PERCEPTION GAMES, THE IMAGE UNDERSTANDING AND INTERPRETATIONAL GEOMETRIES

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The interactive game theoretical approach to the description of perception processes is proposed. The subject is treated formally in terms of a new class of the verbalizable interactive games which are called the *perception games*. An application of the previously elaborated formalism of dialogues and verbalizable interactive games to the visual perception allows to combine the linguistic (such as formal grammars), psycholinguistic and (interactive) game theoretical methods for analysis of the image understanding by a human that may be also useful for the elaboration of computer vision systems. By the way the interactive game theoretical aspects of interpretational geometries are clarified.

The mathematical formalism of interactive games, which extends one of ordinary games [1] and is based on the concept of an interactive control, was recently proposed by the author [2] to take into account the complex composition of controls of a real human person, which are often complicated couplings of his/her cognitive and known controls with the unknown subconscious behavioral reactions. This formalism is applicable also to the description of external unknown influences and, thus, is useful for problems in computer science (e.g. the semi-artificially controlled distribution of resources) and mathematical economics (e.g. the financial games with unknown dynamical factors).

However, the original impetus for the investigations lay in the sphere of human visual perception [3]. When the first steps were made it became clear that it is important to understand this sphere formally including it into the framework of the elaborating interactive game theory. The interactive game theoretical definition of dialogues as psycholinguistic phenomena and the description of the verbalizable interactive games [4] were crucial to make it possible. This article is an attempt to solve the prescribed problem.

I. INTERACTIVE GAMES AND THEIR VERBALIZATION

1.1. Interactive systems and intention fields.

Definition 1 [2]. An *interactive system* (with n *interactive controls*) is a control system with n independent controls coupled with unknown or incompletely known feedbacks (the feedbacks as well as their couplings with controls are of a so complicated nature that their can not be described completely). An *interactive game* is a game with interactive controls of each player.

Below we shall consider only deterministic and differential interactive systems. In this case the general interactive system may be written in the form:

$$(1) \qquad \dot{\varphi} = \Phi(\varphi, u_1, u_2, \dots, u_n),$$

where φ characterizes the state of the system and u_i are the interactive controls:

$$u_i(t) = u_i(u_i^\circ(t), [\varphi(\tau)]|_{\tau \leqslant t}),$$

i.e. the independent controls $u_i^\circ(t)$ coupled with the feedbacks on $[\varphi(\tau)]|_{\tau \leqslant t}$. One may suppose that the feedbacks are integrodifferential on t.

Proposition [2]. *Each interactive system (1) may be transformed to the form (2) below (which is not, however, unique):*

$$(2) \qquad \dot{\varphi} = \tilde{\Phi}(\varphi, \xi),$$

where the magnitude ξ (with infinite degrees of freedom as a rule) obeys the equation

$$(3) \qquad \dot{\xi} = \Xi(\xi, \varphi, \tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n),$$

where $\tilde{u}_i$ are the interactive controls of the form $\tilde{u}_i(t) = \tilde{u}_i(u_i^\circ(t); \varphi(t), \xi(t))$ (here the dependence of $\tilde{u}_i$ on $\xi(t)$ and $\varphi(t)$ is differential on t, i.e. the feedbacks are precisely of the form $\tilde{u}_i(t) = \tilde{u}_i(u_i^\circ(t); \varphi(t), \xi(t), \dot{\varphi}(t), \dot{\xi}(t), \ddot{\varphi}(t), \ddot{\xi}(t), \dots, \varphi^{(k)}(t), \xi^{(k)}(t)))$.

Remark 1. One may exclude $\varphi(t)$ from the feedbacks in the interactive controls $\tilde{u}_i(t)$. One may also exclude the derivatives of ξ and φ on t from the feedbacks.

Definition 2 [2]. The magnitude ξ with its dynamical equations (3) and its contribution into the interactive controls $\tilde{u}_i$ will be called the *intention field*.

Note that the theorem holds true for the interactive games. In practice, the intention fields may be often considered as a field-theoretic description of subconscious individual and collective behavioral reactions. However, they may be used also the accounting of unknown or incompletely known external influences. Therefore, such approach is applicable to problems of computer science (e.g. semi-automatically controlled resource distribution) or mathematical economics (e.g. financial games with unknown factors). The interactive games with the differential dependence of feedbacks are called *differential*. Thus, the theorem states a possibility of a reduction of any interactive game to a differential interactive game by introduction of additional parameters – *the intention fields*.

1.2. Some generalizations. The interactive games introduced above may be generalized in the following ways.

The first way, which leads to the *indeterminate interactive games*, is based on the idea that the pure controls $u_i^\circ(t)$ and the interactive controls $u_i(t)$ should not be obligatory related in the considered way. More generally one should only postulate that there are some time-independent quantities $F_\alpha(u_i(t), u_i^\circ(t), \varphi(t), \dots, \varphi^{(k)}(t))$ for the independent magnitudes $u_i(t)$ and $u_i^\circ(t)$. Such claim is evidently weaker than one of Def.1. For instance, one may consider the inverse dependence of the pure and interactive controls: $u_i^\circ(t) = u_i^\circ(u_i(t), \varphi(t), \dots, \varphi^{(k)}(t))$.

The second way, which leads to the *coalition interactive games*, is based on the idea to consider the games with coalitions of actions and to claim that the interactive controls belong to such coalitions. In this case the evolution equations have the form

$$\dot\varphi = \Phi(\varphi, v_1, \dots, v_m),$$

where v_i is the interactive control of the i-th coalition. If the i-th coalition is defined by the subset I_i of all players then

$$v_i = v_i(\varphi(t), \dots, \varphi^{(k)}(t), u_j^\circ | j \in I_i).$$

Certainly, the intersections of different sets I_i may be non-empty so that any player may belong to several coalitions of actions. Def.1 gives the particular case when $I_i = \{i\}$.

The coalition interactive games may be an effective tool for an analysis of the collective decision making in the real coalition games that spread the applicability of the elaborating interactive game theory to the diverse problems of sociology.

1.3. Differential interactive games and their ε–representations.

Definition 3 [4]. The ε–*representation* of differential interactive game is a representation of the differential feedbacks in the form

(4) $$u_i(t) = u_i(u_i^\circ, \varphi(t), \dots, \varphi^{(k)}(t); \varepsilon_i(t))$$

with the *known* function u_i of all its arguments, where the magnitudes $\varepsilon_i(t) \in \mathcal{E}$ are *unknown* functions of u_i° and $\varphi(t)$ with its higher derivatives:

$$\varepsilon_i(t) = \varepsilon_i(u_i^\circ(t), \varphi(t), \dot\varphi(t), \dots, \varphi^{(k)}(t)).$$

It is interesting to consider several different ε-representations simultaneously. For such simultaneous ε-representations with ε-parameters $\varepsilon_i^{(\alpha)}$ a crucial role is played by the time-independent relations between them:

$$F_\beta(\varepsilon_i^{(1)}, \dots, \varepsilon_i^{(\alpha)}, \dots, \varepsilon_i^{(N)}; u_i^\circ, \varphi, \dots, \varphi^{(k)}) \equiv 0,$$

which are called the *correlation integrals*. Certainly, in practice the correlation integrals are determined *a posteriori* and, thus they contain an important information on the interactive game. Using the sufficient number of correlation integrals one is able to construct various algebraic structures in analogy to the correlation functions in statistical physics and quantum field theory.

1.4. Dialogues as interactive games. The verbalization.

Dialogues as psycholinguistic phenomena can be formalized in terms of interactive games. First of all, note that one is able to consider interactive games of discrete time as well as interactive games of continuous time above.

Defintion 4A (the naïve definition of dialogues) [4]. The *dialogue* is a 2-person interactive game of discrete time with intention fields of continuous time.

The states and the controls of a dialogue correspond to the speech whereas the intention fields describe the understanding.

Let us give the formal mathematical definition of dialogues now.

Definition 4B (the formal definition of dialogues) [4]. The *dialogue* is a 2-person interactive game of discrete time of the form

$$(5) \qquad \varphi_n = \Phi(\varphi_{n-1}, \vec{v}_n, \xi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n).$$

Here $\varphi_n = \varphi(t_n)$ are the states of the system at the moments t_n ($t_0 < t_1 < t_2 < \ldots < t_n < \ldots$), $\vec{v}_n = \vec{v}(t_n) = (v_1(t_n), v_2(t_n))$ are the interactive controls at the same moments; $\xi(\tau)$ are the intention fields of continuous time with evolution equations

$$(6) \qquad \dot{\xi}(t) = \Xi(\xi(t), \vec{u}(t)),$$

where $\vec{u}(t) = (u_1(t), u_2(t))$ are continuous interactive controls with ε–represented couplings of feedbacks:

$$u_i(t) = u_i(u_i^\circ(t), \xi(t); \varepsilon_i(t)).$$

The states φ_n and the interactive controls $\vec{v}_n$ are certain *known* functions of the form

$$(7) \qquad \begin{aligned} \varphi_n &= \varphi_n(\vec{\varepsilon}(\tau), \xi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n), \\ \vec{v}_n &= \vec{v}_n(\vec{u}^\circ(\tau), \xi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n). \end{aligned}$$

Note that the most nontrivial part of mathematical formalization of dialogues is the claim that the states of the dialogue (which describe a speech) are certain "mean values" of the ε–parameters of the intention fields (which describe the understanding).

Important. The definition of dialogue may be generalized on arbitrary number of players and below we shall consider any number n of them, e.g. $n = 1$ or $n = 3$, though it slightly contradicts to the common meaning of the word "dialogue".

An embedding of dialogues into the interactive game theoretical picture generates the reciprocal problem: how to interpret an arbitrary differential interactive game as a dialogue. Such interpretation will be called the *verbalization*.

Definition 5 [4]. A differential interactive game of the form

$$\dot{\varphi}(t) = \Phi(\varphi(t), \vec{u}(t))$$

with ε–represented couplings of feedbacks

$$u_i(t) = u_i(u_i^\circ(t), \varphi(t), \dot{\varphi}(t), \ddot{\varphi}(t), \ldots \varphi^{(k)}(t); \varepsilon_i(t))$$

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is called *verbalizable* if there exist *a posteriori* partition $t_0 < t_1 < t_2 < \ldots < t_n < \ldots$ and the integrodifferential functionals

$$
\begin{aligned}
&\omega_n(\vec{\varepsilon}(\tau), \varphi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n), \\
&\vec{v}_n(\vec{u}^{\circ}(\tau), \varphi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n)
\end{aligned}
$$

(8)

such that

(9)
$$
\omega_n = \Omega(\omega_{n-1}, v_n; \varphi(\tau)|t_{n-1} \leqslant \tau \leqslant t_n).
$$

The verbalizable differential interactive games realize a dialogue in sense of Def.4.

The main heuristic hypothesis is that all differential interactive games "which appear in practice" are verbalizable. The verbalization means that the states of a differential interactive game are interpreted as intention fields of a hidden dialogue and the problem is to describe such dialogue completely. If a differential interactive game is verbalizable one is able to consider many linguistic (e.g. the formal grammar of a related hidden dialogue) or psycholinguistic (e.g. the dynamical correlation of various implications) aspects of it.

During the verbalization it is a problem to determine the moments t_i. A way to the solution lies in the structure of ε-representation. Let the space E of all admissible values of ε-parameters be a CW-complex. Then t_i are just the moments of transition of the ε-parameters to a new cell.

II. Perception games and the image understanding

Let us considered a verbalizable interactive game. We shall suppose for simplicity that the concrete set is finished if some quantity $F(\omega_n, \varphi(t))$ reaches some critical value F_0. The game will be called perception game iff the moments t_i are just the moments of finishing of the concrete sets so the multistage perception game realizes a sequence of sets with initial states coinciding with the final state of the preceeding set. Such construction is not senseless contrary to the most of the ordinary games because the quantity F should be recalculated with the new ω. Thus, we have the following general definition.

Definition 6. The *perception game* is a multistage verbalizable game (no matter finite or infinite) for which the intervals $[t_i, t_{i+1}]$ are just the sets. The conditions of their finishing depends only on the current value of φ and the state of ω at the beginning of the set. The initial position of the set is the final position of the preceeding one.

Practically, the definition describes the discrete character of the perception and the image understanding. For example, the goal of a concrete set may be to perceive or to understand certain detail of the whole image. Another example is a continuous perception of the moving or changing object.

Note that the definition of perception games is applicable to various forms of perception. However, the most interesting one is the visual perception. Besides the numerous problems of human visual perception of reality (as well as of computer vision) there exists a scope of numerous questions of the human behaviour in the computer modelled worlds, e.g. constructed by use of the so-called "virtual reality" (VR) technology. There are no an evident boundary between them because we

can always interpret the internal space of our representations as a some sort of the natural "virtual reality" and apply the analysis of perception in VR to the real image understanding as well as to the activity of imagination. So one should convince that it is impossible to explain all phenomena of our visual perception of reality without deep analysis of its peculiarities in the computer modelled worlds (cf.[5]). Especially crucial role is played by the so-called *integrated realities* (IR), in which the channels only of some kinds of perception are virtual (e.g. visual) whereas others are real (e.g. tactile, kinesthetic).

The proposed definition allows to take into account the dialogical character of the image understanding and to consider the visual perception, image understanding and the verbal (and nonverbal) dialogues together. It may be extremely useful for the analysis of collective perception, understanding and controlling processes in the dynamical environments – sports, dancings, martial arts, the collective controlling of moving objects, etc.

On the other hand this definition explicates the self-organizing features of human perception, which may be unraveled by the game theoretical analysis.

And, finally, the definition put a basis for a systematical application of the linguistic (e.g. formal grammars) and psycholinguistic methods to the image understanding as a verbalizable interactive game with a mathematical rigor.

Also interpreting perception processes and the image understanding as the verbalizable interactive games we obtain an opportunity to adapt some procedures of the image understanding to the verbalizable interactive games of a different nature, e.g. to the verbal dialogues. It may enlight the processes of generation of subjective figurative representations, which is important for the analysis of the understanding of speech in dialogues.

Traditionally the problems of the visual perception are related to geometry (as descriptive as abstract) so it is reasonable to pay an attention to the geometrical background for the perception videogames and then to combine both geometrical and interactive game theoretical approaches to the visual perception and the image understanding.

III. Interpretational geometries, intentional anomalous virtual realities and their interactive game theoretical aspects

3.1. Interpretational figures [6]. Geometry described below is related to a class of interactive information systems. Let us call an interactive information system computer graphic (or interactive information videosystem) if the information stream "computer–user" is organized as a stream of geometric graphical data on a screen of monitor; an interactive information system will be called psychoinformation if an information transmitted by the channel "user–computer" is (completely or partially) subconscious. In general, an investigation of interactively controlled (psychoinformation) systems for an experimental and a theoretical explication of possibilities contained in them, which are interesting for mathematical sciences themselves, and of "hidden" abstract mathematical objects, whose observation and analysis are actually and potentially realizable by these possibilities, is an important problem itself. So below there will be defined the notions of an interpretational figure and its symbolic drawing that undoubtly play a key role in the description of a computer–geometric representation of mathematical data in interactive information systems. Below, however, the accents will be focused a bit more on

applications to informatics preserving a general experimentally mathematical view, the interpretational figures (see below) will be used as pointers to droems and interactive real-time psychoinformation videosystems will be regarded as components of integrated interactive videocognitive systems for accelerated nonverbal cognitive communications.

In interactive information systems mathematical data exist in the form of an interrelation between the geometric internal image (figure) in the subjective space of the observer and the computer-graphic external representation. The latter includes visible (drawings of the figure) and invisible (analytic expressions and algorithms for constructing these images) elements. Identifying geometric images (figures) in the internal space of the observer with computer-graphic representations (visible and invisible elements) is called a *translation*, in this way the visible object may be not identical with the figure, so that separate visible elements may be considered as modules whose translation is realized independently. The translation is called an *interpretation* if the translation of separate modules is performed depending on the results of the translation of preceding ones.

Definition 7. The figure obtained as a result of interpretation is called an *interpretational figure.*

Note that the interpretational figure may have no usual formal definition; namely, only if the process of interpretation admits an equivalent process of compilation definition of the figure is reduced to definitions of its drawings that is not true in general. So the drawing of an interpretational figure defines only dynamical "technology of visual perception" but not its "image", such drawings will be called *symbolic.*

The computer-geometric description of mathematical data in interactive information systems is closely connected with the concept of anomalous virtual reality.

3.2. Intentional anomalous virtual realities [3,6].

Definition 8. (A). *Anomalous virtual reality (AVR) in a narrow sense* means some system of rules of a nonstandard descriptive geometry adapted for realization on videocomputers (or multisensorial systems of "virtual reality"). *Anomalous virtual reality in a wide sense* also involves an image in cyberspace formed in accordance with said system of rules. We shall use the term in its narrow sense. **(B).** *Naturalization* is the constructing of an AVR from some abstract geometry or physical model. We say that anomalous virtual reality *naturalizes* the abstract model and the model *transcendizes* the naturalizing anomalous virtual reality. **(C).** *Visualization* is the constructing of certain image or visual dynamics in some anomalous virtual reality (realized by hardware and software of a computer-grafic interface of the concrete videosystem) from the objects of an abstract geometry or processes in a physical model. **(D).** Anomalous virtual reality, whose objects depend on the observer, is called an *intentional anomalous virtual reality (IAVR)*. The generalized perspective laws for IAVR contain the interactive dynamical equations for the observed objects in addition to standard (geometric) perspective laws. In IAVR the observation process consists of a physical process of observation and a virtual process of intentional governing of the evolution of images in accordance with the dynamical perspective laws.

In intentional anomalous virtual reality (IAVR) that is realized by hardware and software of the *computer-graphic interface of the interactive videosystem* being geo-

metrically modelled by this IAVR (on the level of descriptive geometry whereas the model transcendizing this IAVR realizes the same on the level of abstract geometry) respectively, the observed objects are demonstrated as connected with the observer who acts on them and determines, or fixes, their observed states so that the objects are thought only as a potentiality of states from the given spectrum whose realization depends also on the observer. The symbolic drawings of interpretational figures may be considered as states of some IAVR.

Note that mathematical theory of anomalous virtual realities (AVR) including the basic procedures of naturalization and thanscending connected AVR with the abstract geometry is a specific branch of *modern nonclassical descriptive (computer) geometry.*

Definition 8E. The set of all continuously distributed visual charcteristics of the image in anomalous virtual reality is called an *anomalous color space*; the anomalous color space elements of noncolor nature are called *overcolors*, and the quantities transcendizing them in an abstract model are called *"latent lights"*. The set of the generalized perspective laws in a fixed anomalous color space is called a *color-perspective system.*

3.3. Remarks on the interactive game theoretical aspects. Certainly, the interpretational geometries may be considered as the perception games. An interesting geometrical consequence of such approach was proposed [7].

Proposition. *There exist models of interpretational geometries in which there are interpretational figures observed only in a multi-user mode.*

It seems that this proposition may be regarded as a startpoint for the future interactions between geometry and interactive game theory in the sphere of mathematical foundations for the collective perception and image understanding of real objects as well as objects in the computer VR or IR systems.

IV. Conclusions

Thus, the interactive game theoretical approach to the description of perception processes is proposed. A new class of the multistage verbalizable interactive games, the perception games, is introduced. The interactive game theoretical aspects of interpretational geometries are clarified. Perspectives are sketched.

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